Engineering Notes

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The Effects of Pitching-Moments on Phugoid and Height Mode in Supersonic Flight

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Nomenclature†

AR	= aspect ratio
a_h	= $(1/a)da/dh$, relative gradient of speed of sound
$C_{mh}{}^*$	= effective pitching moment derivative with respect to height change (compressibility-, aeroelasticity-, thrust-offset-effects, Ref. 2)
$C_{mV}{}^*$	= effective pitching moment derivative with respect to speed change (compressibility-, aeroelasticity-, thrust-offset-effects, Ref. 2)
$\partial C_D/\partial C_L$	= $C_{D\alpha}/C_{L\alpha}$
h	= height
i_y	= radius of gyration, $\bar{i}_y = (i_y/\bar{c})^2$
k	= $1/(2C_L)\ \partial C_D/\partial C_L$
M	= Mach number
$n_V{}^*$	= denoting effective thrust change ΔT due to speed change ΔV, including compressibility effects ($\Delta T/T = n_V{}^* \Delta V/V$, Ref. 2)
$n_\rho{}^*$	= denoting effective thrust change ΔT due to height change Δh, including compressibility and temperature effects ($\Delta T/T = n_\rho{}^* \rho_h \Delta h$, Ref. 2)
$s_{1,2\ldots5}$	= roots of the characteristic equation
β_V	= $2 - M^2/(M^2-1)$
β_ρ	= $1 + (a_h/\rho_h)M^2/(M^2-1)$
ρ_h	= $(1/\rho)\ d\rho/dh$ relative density gradient
σ_P	= damping of phugoid mode
ω_{nP}/ω_P	= natural frequency/frequency of phugoid mode

ATMOSPHERIC changes with altitude and compressibility have significant effects on the longitudinal motion in high-speed flight.[3-6] This is especially true in the supersonic region.[7] The stability of the motion in supersonic flight, where maintaining a given altitude is of increased importance, can be strongly influenced by the pitching-moment characteristics of the aircraft. The purpose of this Note is to give an explicit expression of the effects of pitching-moments on the phugoid and height modes, and thus to show their consequences for altitude stability in general.

Taking the effects of atmospheric changes with altitude into consideration, the characteristic equation of the linearized equations-of-motion for longitudinal flight is of the fifth order. On the assumption that the magnitude of the roots is given by

$$|s_{1,2}| \gg |s_{3,4}| \gg |s_5| \tag{1}$$

the approximate factorization of the characteristic equation yields[2] for the height mode

$$s_5 \approx \frac{V}{\bar{c}} \frac{(\beta_V n_\rho{}^* - \beta_\rho n_V{}^*)C_D\bar{c}\rho_h + (C_{L\alpha}/C_{m\alpha})(a_1 C_{mh}{}^* + a_2 C_{mV}{}^*)}{\beta_V C_L - \beta_\rho \mu\bar{c}\rho_h + (C_{L\alpha}/C_{m\alpha})(C_{mh}{}^*/C_L - C_{mV}{}^*)} \tag{2}$$

where

$$a_1 = (1/\mu)[\beta_V(\partial C_D/\partial C_L - C_D/C_L) + n_V{}^* C_D/C_L]$$
$$a_2 = -\bar{c}\rho_h[\beta_\rho(\partial C_D/\partial C_L - C_D/C_L) + n_\rho{}^* C_D/C_L]$$

The natural frequency of the phugoid mode is given by

$$\omega_{nP}{}^2 \approx (g/\bar{c})a_3 C_{m\alpha}[\beta_V C_L - \beta_\rho\mu\bar{c}\rho_h + (C_{L\alpha}/C_{m\alpha})(C_{mh}{}^*/C_L - C_{mV}{}^*)] \tag{3}$$

where $a_3 = 2/(2\mu C_{m\alpha} + C_{L\alpha}C_{mq})$

The damping of the phugoid mode can be expressed as

$$\sigma_P = \sigma_{P0} + \Delta\sigma_{P1} + \Delta\sigma_{P2} - s_5/2 \tag{4a}$$

σ_{P0}, which can be approximated by

$$\sigma_{P0} \approx V/(2\mu\bar{c})[(n_V{}^* - \beta_V)C_D + a_3\beta_V C_{L\alpha}C_L{}^2(C_{mq}(k - a_3(C_{mq} + C_{m\dot{\alpha}})/4) - a_3\bar{i}_y\mu C_{m\alpha})] \tag{4b}$$

denotes the case of constant atmosphere with $C_{mV}{}^* = 0$. $\Delta\sigma_{P1}$ and $\Delta\sigma_{P2}$, in combination with s_5, show the effects of atmospheric changes with altitude as well as the effects of $C_{mV}{}^*$ and $C_{mh}{}^*$. They are given by

$$\begin{aligned} \Delta\sigma_{P1} &\approx (V/2)a_3\rho_h C_L[\bar{i}_y(\beta_\rho n_V{}^* - \beta_V n_\rho{}^*)C_D + a_3\beta_\rho C_{L\alpha}(\bar{i}_y\mu C_{m\alpha} + C_{mq}(C_{mq} + C_{m\dot{\alpha}})/4)] \\ \Delta\sigma_{P2} &\approx (V/\bar{c})a_3 C_{L\alpha}C_L[kC_{mV}{}^* + (C_{mq} + C_{m\dot{\alpha}} - 2\bar{i}_y C_{L\alpha})(C_{mh}{}^*/C_L - C_{mV}{}^*)a_3/4] \end{aligned} \tag{4c}$$

Considering first the case $C_{mV}{}^* = C_{mh}{}^* = 0$, it follows that the height mode is mainly a result of thrust characteristics [Eq. (2)], i.e.,

$$s_5 \propto \beta_V n_\rho{}^* - \beta_\rho n_V{}^*$$

The pitching-moments due to $C_{m\alpha}$, C_{mq} and $C_{m\dot{\alpha}}$ have no effect on the height mode s_5. If $|2\mu C_{m\alpha}| \gg |C_{L\alpha}C_{mq}|$, their effect on the natural frequency ω_{nP} of the phugoid mode is negligible. Increased $|C_{m\alpha}|$ adds to phugoid damping if[2]

$$|C_{m\alpha}| > |C_{mq}|/(2\mu)[(2\bar{i}_y C_{L\alpha} - C_{mq} - C_{m\dot{\alpha}})/(\bar{i}_y - (k/a_4)C_{mq})] \tag{5}$$

where $a_4 = 1 - (\mu\bar{c}\rho_h/C_L)(\beta_\rho/\beta_V)$

This can be assumed for normal flight conditions in high altitude. In contrast, increased pitch damping C_{mq} adds to phugoid damping only if

$$|C_{m\alpha}| < 1/(2k\mu)[a_4(2\bar{i}_y C_{L\alpha} - C_{mq} - C_{m\dot{\alpha}}/2) + kC_{L\alpha}C_{mq}] \tag{6}$$

Figure 1 shows these effects for an airplane (referred to as "basic airplane") the characteristics of which are given by: $S = 460\ m^2$, $\bar{c} = 15\ m$, $AR = 2$, $m = 1.4 \cdot 10^5$ kg, $i_y = 10\ m$, $h = 21{,}000m$, $M = 3$, $C_D/C_L = \frac{1}{8}$, $C_{L\alpha} = 1.55$, $k = 1/(0.35 \cdot \pi \cdot AR)$, $C_{m\alpha} = -0.155$, $C_{mq} = 3 \cdot C_{m\dot{\alpha}} = -\pi/3$, $n_V{}^* = 2$, $n_\rho{}^* = 1$.

The pitching-moments due to speed and height changes ($C_{mV}{}^*$ and $C_{mh}{}^*$) have significant effects on the phugoid and height modes. In general, $C_{mV}{}^* < 0$ and/or $C_{mh}{}^* > 0$ can lead to large values of s_5 by decreasing the denominator in Eq. (2). This has strong consequences with regard to stability or instability, depending on the factors $a_{1,2}$ in the nominator of Eq. (2), which are determined by the drag and thrust characteristics of the airplane. In the opposite case ($C_{mV}{}^* > 0$ and/or $C_{mh}{}^* < 0$), changes of s_5 may be less pronounced. It is important that the effect of $C_{mh}{}^*$ in the denominator, as compared with that of $C_{mV}{}^*$, increases with the square of airspeed.

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† All symbols not explained correspond to the notation of Ref. 1.

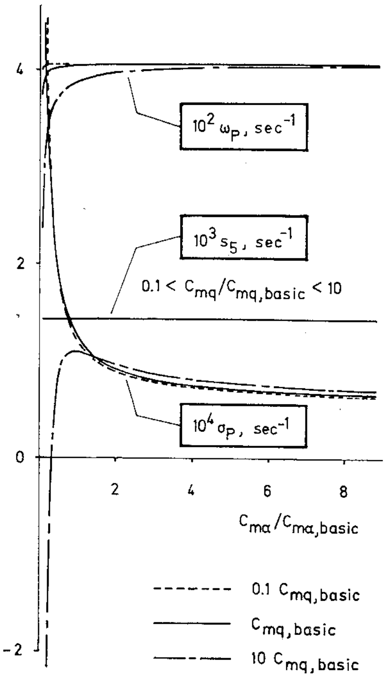

Fig. 1 Effect of $C_{m\alpha}$ and C_{mq} on phugoid and height mode ($C_{mV}{}^* = C_{mh}{}^* = 0$).

With regard to $C_{m\alpha}$, it follows from Eq. (2) that increase of $|C_{m\alpha}|$ reduces the effects of both $C_{mV}{}^*$ and $C_{mh}{}^*$.

The example of Fig. 2 shows s_5 as a function of $C_{mV}{}^*$ and $C_{mh}{}^*$ where $C_{m\alpha}$, $n_V{}^*$ and $n_\rho{}^*$ have been introduced as parameters to illustrate the effects discussed above. In particular, the variation of $n_\rho{}^*$ and the superposition of $C_{mV}{}^*$ and $C_{mh}{}^*$ show the susceptibility of s_5 [i.e. the nominator in Eq. (2)] to parameter changes in the region $C_{mh}{}^* > 0$. Here, strong stability of the basic airplane is converted into strong instability.

As to the natural frequency ω_{nP} of the phugoid mode, it follows from Eq. (3) that ω_{nP} is increased by either $C_{mV}{}^* > 0$ or $C_{mh}{}^* < 0$ and is decreased in the opposite case. Here again, $C_{m\alpha}$ reduces the influence of $C_{mV}{}^*$ and $C_{mh}{}^*$.

The effects of $C_{mV}{}^*$ and $C_{mh}{}^*$ on phugoid damping can be divided into two components, the first being the inverse of $s_5/2$ and the second the term $\Delta\sigma_{P2}$ [Eq. (4c)]. $\Delta\sigma_{P2}$ always adds to damping for $C_{mh}{}^* > 0$. In case of $C_{mV}{}^* > 0$, it adds to damping when $|\mu C_{m\alpha}|$ is sufficiently large. On the other hand, the effects of $C_{mV}{}^*$ and $C_{mh}{}^*$ due to the s_5-component vary depending on the factors shown above. The inverse s_5-contribution is dominating when $|s_5|$ is large. This is very important since all effects stabilizing the height mode add to phugoid instability and vice versa. These effects on phugoid roots are illustrated in Fig. 3. Comparison with Fig. 2 shows the significance of the s_5-contribution to phugoid damping when $|s_5|$ is large.

With regard to speed feedback to thrust, there are also important consequences of atmospheric changes with altitude.[2] Phugoid damping is significantly increased by negative feedback in low speed flight,[8,9] where the height mode can be ignored. In high-speed flight, however, improvement of phugoid damping is strongly reduced in favor of stabilizing the height mode. In this case, where

$$C_L/(\mu\bar{c}) = g/V^2 \ll |\rho_h| \ll 1/\bar{c},$$

the changes Δs_5 and $\Delta\sigma_p$, due to a change $\Delta n_V{}^*$ (denoting speed feedback to thrust), can be approximated by

$$\Delta s_5(\Delta n_V{}^*) \approx V/(\mu\bar{c})[1 + (\beta_V/\beta_\rho)C_L/(\mu\bar{c}\rho_h)]C_D \Delta n_V{}^* \qquad (7)$$

and

$$\frac{\Delta\sigma_P(\Delta n_V{}^*)}{\Delta\sigma_{P0}(\Delta n_V{}^*)} \approx \left[\frac{2\bar{i}_y\beta_\rho\bar{c}\rho_h}{2C_{m\alpha} + C_{L\alpha}C_{mq}/\mu} - \frac{\beta_V/\beta_\rho}{\mu\bar{c}\rho_h}\right] C_L \qquad (8)$$

(see Eqs. (2) and (4) with $C_{mV}{}^* = C_{mh}{}^* = 0$). From this it follows that

$$\Delta s_5(\Delta n_V{}^*) \approx 2\,\Delta\sigma_{P0}(\Delta n_V{}^*) \qquad (9)$$

and

$$|\,\Delta\sigma_P(\Delta n_V{}^*)/\Delta\sigma_{P0}(\Delta n_V{}^*)\,| \ll 1 \qquad (10)$$

(In low-speed flight, the actual phugoid damping $\Delta\sigma_P(\Delta n_V{}^*)$ is approximately equal to the reference value of constant atmosphere $\Delta\sigma_{P0}(\Delta n_V{}^*)$, i.e. $\Delta\sigma_P(\Delta n_V{}^*)/\Delta\sigma_{P0}(\Delta n_V{}^*) \approx 1$. Here, s_5 as well as $\Delta s_5(\Delta n_V{}^*)$ can be ignored.) The $\Delta n_V{}^*$-effect on phugoid damping $\Delta\sigma_P(\Delta n_V{}^*)$ is more stabilizing when $C_{mV}{}^* > 0$. On the contrary, when

$$C_{mV}{}^* < \beta_V C_L C_{m\alpha}/C_{L\alpha}$$

negative speed feedback even reduces phugoid damping. $C_{mh}{}^* < 0$ does not alter Eqs. (9) and (10) essentially, whereas in case of $C_{mh}{}^* > 0$, the $\Delta n_V{}^*$-effect on $\Delta\sigma_P(\Delta n_V{}^*)$ is increased.

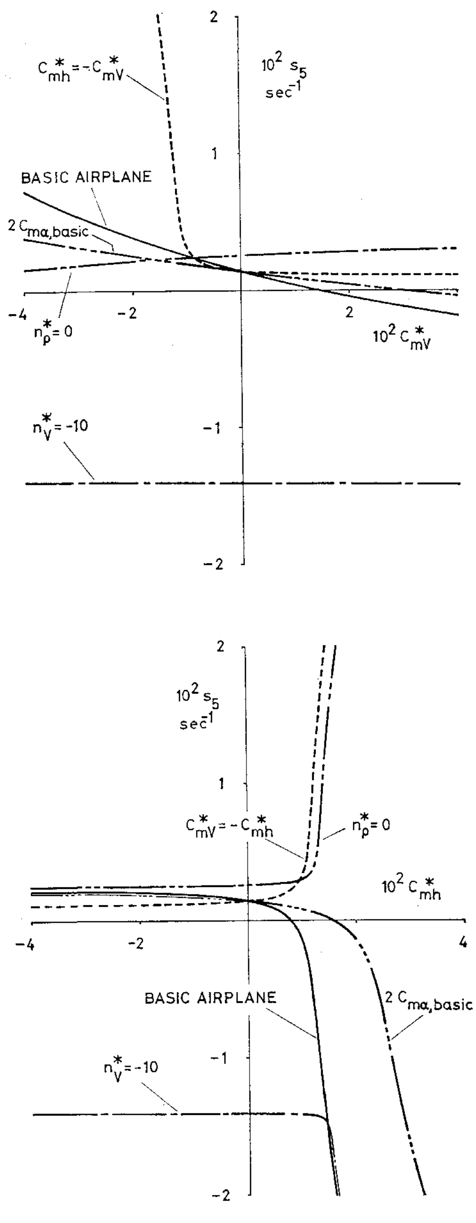

Fig. 2 Effect of $C_{mV}{}^*$ and $C_m{}^*$ on height mode.

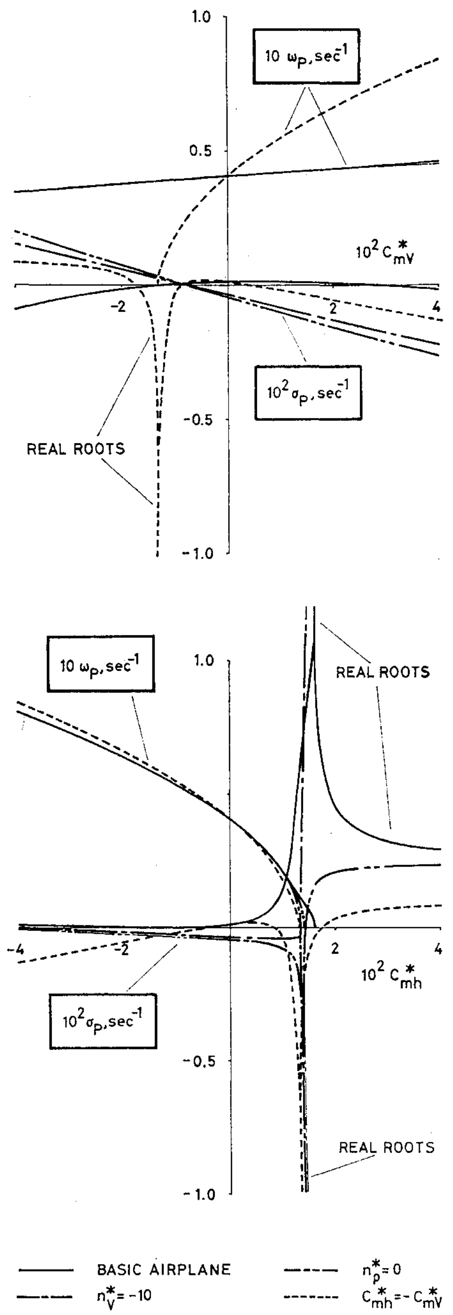

Fig. 3 Effect of $C_{mV}{}^*$ and $C_{mh}{}^*$ on phugoid mode.

It becomes very strong when $C_{mh}{}^* \to \beta_\rho \mu \bar{c}_h \rho_h C_L C_{m\alpha}/C_{L\alpha}$. These effects corresponding to a variation of $n_V{}^*$ are also shown in Figs. 2 and 3 where $n_V{}^* = -10$ is equivalent to $\Delta n_V{}^* = -12$.

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Response of a Trailing Vortex to Axial Injection into the Core

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Introduction

TRAILING vortices behind lifting surfaces create serious problems; e.g., aeroelastic loads and noise in the case of helicopter blades, and turbulent gusts for airplanes following large aircraft. The severity of these problems is determined by the intensity and persistence of the vortices. A simple method of quickly and effectively dispersing the vorticity in these vortex cores without penalizing the performance has been the object of several investigations.[1–3] Various methods like the use of swept and porous tips (Ref. 1), twisted swept tips (Ref. 2) and vortex dissipators (small spanwise fences placed at the wing tip on the suction side—Ref. 3) have been tried with varying success. This Note describes briefly some preliminary qualitative results of an investigation into the effects of axial air injection into a vortex core. A more detailed account appears in Ref. 4.

Other investigations[5–8] on the effect of injection on vortex flows have appeared recently. Rinehart et al.[5] used a set-up essentially similar to the present one and their photographs of the vortex obtained by the hydrogen bubble technique and vorticity contours indicate that injection can indeed spread out the vorticity concentrated in the core. However his theory[6] indicates that the phenomenon is governed by the mass-flow-rate of injection contrary to our evidence presented herein that it is governed by the momentum-rate. Poppleton's investigations (Ref. 7) though not exactly on a trailing vortex core (his experiments were conducted on a swirling flow in a tube) confirm the fact that injection is effective. His measurements clearly show decreases in peak tangential velocities and increases in the size and turbulence level in the core, and the axial decay rate of tangential velocities. Monnerie and Tognet[8] have also investigated the effect of injection, not axially but vertically down along the chord at the wing tip.

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